# Further Maths Mechanics Notes

## Momentum and Impulse

Momentum $= mv$ (units $=$ Ns or $Kgms^{-1}$)

Impulse $= Ft = mv - mu = \Delta$ momentum

Cons. momentum: total momentum before impact = total momentum after impact

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

These can be written as vectors: $\underline{I} = m\underline{v} - m\underline{u}$

$$m_1 \underline{u_1} + m_2 \underline{u_2} = m_1 \underline{v_1} + m_2 \underline{v_2}$$

## Work, energy and power

Work done = component of force in direction of motion × distance moved in direction of force

$W = Fd = (Fd\cos\theta)$

Work against gravity $= mgh$ $\quad KE = \frac{1}{2}mv^2 \quad P.E = mgh$

$W =$ change in total energy

Before calculating a particle's potential energy you must choose a zero level

### Principle of conservation of mechanical energy

↳ when no external forces other than gravity do work on a particle during its motion, the sum of the particle's kinetic and potential energy remains constant

Power is the rate of doing work: $P = E/t \quad P = Fv$

## Elastic strings and springs

When an elastic string or spring is stretched the tension $T$ is proportional to the extension $x$

$\lambda$ has units $N$

$T \propto x \Rightarrow T = kx$ ⟸ This is known as Hooke's Law.

The constant $k$ depends on the length of the spring or string $L$ and the modulus of elasticity, $\lambda$

$T = \frac{\lambda x}{L}$ $\left(k = \frac{\lambda}{L}\right)$. The area under a force-distance graph is the work done

Work done in stretching a string or spring from its natural length $L$ to $(L+x)$ is: $W = \frac{\lambda x^2}{2L}$

When no external forces act (other than gravity) then the sum of a particle's KE, GPE and EPE is constant.

## Elastic collisions in one dimension

Newton's Law of restitution: $e = \dfrac{\text{speed of separation of particles}}{\text{speed of approach of particles}}$

coefficient of restitution: $0 \le e \le 1$

These are the same

For a collision with a smooth plane: $e = \dfrac{\text{speed of rebound}}{\text{speed of approach}}$

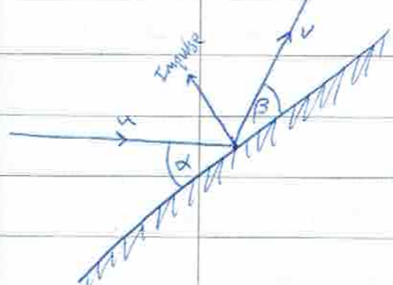
The loss of kinetic energy due to impact is:

$$KE\ loss = \left(\tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2\right) - \left(\tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2\right)$$

= kinetic energy before − kinetic energy after

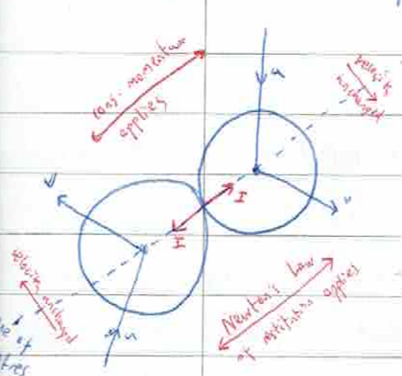
## Elastic collisions in two dimensions

In an oblique impact between a smooth sphere and a smooth fixed surface:




- The impulse on the sphere acts perpendicular to the surface, through the sphere's centre
- The component of the velocity of the sphere parallel to the surface is unchanged
  $v\cos\beta = u\cos\alpha$
- You can use Newton's law of restitution to find the component of the velocity of the sphere perpendicular to the surface
  ↳ $v\sin\beta = eu\sin\alpha$

In an impact between two spheres:




- The reaction between the two spheres acts along the line of centres, so also the impulse affecting each sphere acts along the line of centres
- The components of the velocities of the spheres perpendicular to the line of centres are unchanged by the impact
- Newton's law of restitution applies to the components of the velocities of the spheres parallel to the line of centres.
- The principle of conservation of momentum applies parallel to the line of centres